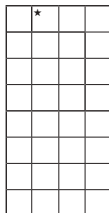
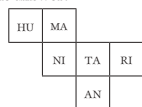


**Magic squares:** Record solutions in this answer box. Begin in the starred box; proceed left to right in each row. Answers form two magic squares, a  $3 \times 3$  and a  $4 \times 4$  array. Remaining answers pay tribute to a master of recreational mathematics.



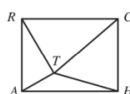
The six squares in the diagram are to be folded into a cube. A humanitarian wants to know which side will be opposite the side labeled AN. Can you help him determine the answer?



1

CHAR is a rectangle with point T in its interior. Determine which line segment in the diagram has the square of its measure equal to the expression

$$(AT)^2 - (HT)^2 + (CT)^2.$$



2

Each letter of the alphabet is assigned an integer, starting with A = 0, B = 1, and so on. The numbers repeat after every seven letters, so that G = 6, H = 0, and I = 1, continuing on to Z. What two-letter word is represented by the digits 16?

3

The graphs of the equations

$$y = 3^x \text{ and } y = 12x - 9$$

intersect. Find the sum of the  $x$ -values for all points of intersection.

4

Only 2 two-digit integers are equal to three times the product of their digits. Find the absolute value of the difference of these 2 integers.

5

Evaluate

$$\left( \frac{2011}{1006} \right) = \frac{2011 C_{1006}}{2010 C_{1005}}$$

to the nearest integer.

6

GUIDANCE is a regular octagon with center O. We define  $R_1$ ,  $R_2$ , and  $R_3$  as counter-clockwise rotations of, respectively,  $45^\circ$ ,  $90^\circ$ , and  $135^\circ$  about O. Which diagonal will be mapped onto  $\overline{CI}$  if each rotation is applied exactly once?

7

A puppy weighs  $3/4$  of its weight plus  $3/4$  lb. How many pounds does the puppy weigh?

8

In a group of 50 girls, each girl is either blonde or brunette, and each has either blue or brown eyes. Find the number of brown-eyed blondes if 14 girls are blue-eyed blondes, 31 are brunettes, and 18 have brown eyes.

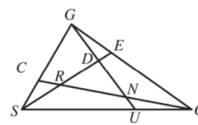
9

Compute the digit in the thousands place when calculating the following sum:

$$6 + 66 + 666 + 6666 + \dots + 6,666,666,666$$

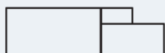
10

The points C, E, and U are the  $1/3$  points on the respective sides of  $\triangle SGO$ . Which triangle in the figure has exactly  $1/7$  of the area of  $\triangle SGO$ ?



11

Three similar rectangles with integral widths and lengths in the ratio 1:2 form a nonconvex hexagon, as shown. The width of the largest rectangle is 3. Find the sum of the digits of the hexagon's perimeter.



12

The equation  $x^2 + px + q = 0$ ,  $q \neq 0$ , has two unequal roots such that the squares of the roots are the same as the two roots. Calculate the product  $p \cdot q$ .

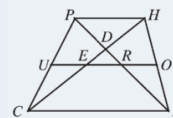
13

A rectangular sheet of paper 2 in.  $\times$  in.,  $x > 2$ , is cut into three congruent pieces by two straight cuts, as shown. Find the number of sq. in. in the area of the original rectangle.



14

In trapezoid CPHL with median OU,  $CL = 20$  and  $PH = 12$ . Of the many segments in the figure, which one must have length 4?



15

Find all integral values of  $x$  such that

$$2^{2x^2-9x+4} = 5^{x^2-x-12}.$$

16

The repeating decimal 0.3333... in base 10 is the fraction  $1/3$ . The same repeating decimal in base 9 can be written in reduced form as  $a/b$  where  $a$  and  $b$  are in base 10. Find the sum

$$a + b.$$

17

The expression

$$x^4 + 11x^3 + 30x^2 + 53x - 7$$

factors into two nonfactorable quadratics with integral coefficients. Find the sum of all the coefficients of the quadratics.

18

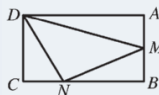
Find the value of  $k$  such that the graphs of the lines defined by the three equations

$$\begin{aligned} 14x + 3y &= 9, \\ 9x - 2y &= 49, \text{ and} \\ -7x - 6y &= k \end{aligned}$$

are concurrent.

19

The area of the rectangle shown is 48. The midpoint of  $\overline{AB}$  is M, and N is  $1/3$  the distance from C to B. Compute the area of  $\triangle DNM$ .



20

In 1941 Fred's children were both between the ages of 10 and 20. The sum of the cube of one child's age and the square of the other child's age gives the year in which Fred's wife was born. How old was his wife in 1941?

21

Compute the maximum value for the following function:

$$f(x) = 3\sin(3x) + 4\cos(3x)$$

22

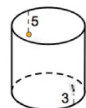
The perimeter of a sector of a circle is 40. Find the radius of the sector that maximizes its area.

23

A teacher bought 100 puzzles for \$82.90 to give out to the members of her math team. She bought three different types of puzzle. Type A cost 40¢ each, type B cost 70¢ each, and type C cost \$1 each. How many more type C puzzles than type A puzzles did she buy?

24

An ant on the outside of a glass, 3 in. from the bottom, sees a drop of honey inside the glass, 5 in. from the top and exactly half way around the glass. The glass is 13 in. high and has a circumference of 16 in. Compute the length of the shortest path for the ant to reach the honey.



25

Given that the three quantities

$$\begin{aligned} \log_2(\log_3(\log_4(a))), \\ \log_5(\log_6(\log_7(b))), \text{ and} \\ \log_7(\log_8(\log_9(c))) \end{aligned}$$

are all equal to 0, find

$$\frac{a}{b} + c.$$

26

The fraction

$$\frac{30}{\sqrt{3} + \sqrt{5} + \sqrt{8}}$$

can be written as

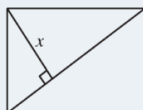
$$p\sqrt{3} + q\sqrt{5} + r\sqrt{8} + s\sqrt{30}.$$

Find the value of the expression

$$p + q + r + s.$$

27

The rectangle shown in the diagram has width 15 and length 20. Find  $x$ , the length of the segment perpendicular to the diagonal.



28

In base  $b$ , the number 331<sub>b</sub> is the square of an integer in base 10. Compute the smallest positive value of  $b$ .

29

A bicycle storeowner asks one of his employees to count the number of bicycles and tricycles in his store. The employee knows that the owner likes puzzles, so he tells him that there are a total of 169 wheels but only 152 pedals. How many more bicycles than tricycles are in the store?

30

Compute the positive geometric mean for the set of positive divisors of the number 324.

31